

# Development of a Complexity Framework and Metrics for Assembly Supply Chain Topologies

**Mrs.N.Prashanthi**

*Assistant Professor, Department of H&S,  
Malla Reddy College of Engineering for Women.,  
Maisammaguda., Medchal., TS, India*

## Article Info

Received: 30-03-2023

Revised: 16-04-2023

Accepted: 29-04-2023

## Abstract:

In this paper, we present a methodological framework for conceptual modeling of assembly supply chain (ASC) networks. Models of such ASC networks are divided into classes on the basis of the numbers of initial suppliers. We provide a brief overview of select literature on the topic of structural complexity in assembly systems. Subsequently, the so called Vertex degree index for measuring a structural complexity of ASC networks is applied. This measure, which is based on the Shannon entropy, is well suited for the given purpose. Finally, we outline a generic model of quantitative complexity scale for ASC Networks.

## Introduction

Assembly supply chain (ASC) systems are becoming increasingly complex due to technological advancements and the use of geographically diverse sources of parts and components. One of the major challenges at the early configuration design stage is to make a decision about a suitable networked manufacturing structure that will satisfy the production functional requirements and will make managerial tasks simpler and more cost effective. In this context any reduction of redundant complexity of ASC is considered as a way to increase organizational performance and reduce operational inefficiencies. Furthermore, it is known that higher complexity degree of ASC systems makes it difficult to manage material and information flows from suppliers to end-users, because a small changes may lead to a massive reaction. Nonlinear systems that are unpredictable cannot be solved exactly and need to be approximated. One way to approximate complex dynamic systems is to transform them into static structural models that could be evaluated

with graph-based methods. Thus, structural complexity approaches that assess topological properties of networks are addressed in this paper.

Structural complexity theory is a branch of computational complexity theory that aims to evaluate systems' characteristics by analysing their structural design. In structural complexity the main focus is on complexity classes, as opposed to the study of systems behaviour to be conducted more efficiently. According to Hartman is [1]: "structural complexity investigates both internal structures of complexity classes, and relations that hold between different complexity classes". In this study our main intent is to identify topological classes of assembly supply chains (ASC). Our approach to generate classes of ASCs is based on some specific rules and logical restrictions described in Section 3. Subsequently, in Section 4, we present a method to compute the structural complexity of such networks. Finally, in the Conclusions section, the main contributions of our paper are mentioned.

## Related Works

Complexity theory has captured the attention of the scientific community across the World and its proponents tout it as a dominant scientific trend [2]. According to ElMaraghy et al. [3], increasing complexity is one of the main challenges facing production companies. Complexity of systems has been defined in several ways because it has many aspects depending and on the viewpoint and context in which a system is analysed. For

example, Kolmogorov complexity [4,5] is based on algorithmic information theory, which is related to Shannon entropy [6]. Both theories use the same unit—the bit—for measuring information. Shannon's entropy has been generalized in different directions. For example, it has been widely used in biological and ecological networks [7–9].

Information theories consider information complexity as the minimum description size of a system [10–12]. Related pertinent findings with regards to the impact of organization size on increasing differentiation have been expressed in the literature [13–15]. These authors maintain that increasing the differentiation of networks creates a control problem of integrating the differentiated subunits. According to Strogatz [16], the most basic issues in the study of complex networks are structural properties because structure always affects function. Moreover, he adds that there are missing unifying principles underlying their topology. The lack of such principles makes it difficult to evaluate of certain topological aspects of networks, including complexity. Structural or static complexity characteristics [17,18] are related to the fixed nature of products, hierarchical structures, processes and intensity of interactions between functionally differentiated subunits. So-called 'layout complexity' in this context is studied that has a significant impact on the operation and performance of manufacturing systems [19]. Hasan et al. [20] argue that "a good layout contributes to the overall efficiency of operations and can reduce by up to 50% the total operating expenses". On the other hand, experiences show that managers prefer to continue with the inefficiencies of existing layouts rather than undergo expensive and time consuming layout redesign.

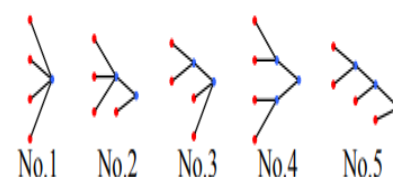
The relationship between product variety and manufacturing complexity in assembly systems and supply chains has been investigated by several authors [21–23]. Morse and You [24] developed the method called Gap Space to analyse assembly

success in terms of non/interference of components. Zhu et al. [25] proposed a complexity measure based on quantifying human performance in manual mixed-model assembly lines where operators have to make choices for various assembly activities. An original approach to assessment of overall layout complexity was developed by Samy [26]. He proposed an overall Layout Complexity Index (LCI) which combines several indices. Obviously, there are many other research articles related to the topic of our paper. Based on a previous analysis of the literature sources it is possible to say that there are several aspects by which one could examine assembly supply chain complexity. In this paper, we propose to compute structural complexity with reduced effort using standardized classes of supply chain networks.

### Generating of Assembly Supply Chain Classes

An assembly-type supply chains is one in which each node in the chain has at most one successor, but may have any number of predecessors. Such supply chain structures are convergent and can be divided into two types, modular and non-modular. In the modular structure, the intermediate sub assemblers are understood as assembly modules, while the non-modular structure consists only from suppliers (initial nodes) and a final assembler (end node). The framework for creating topological classes of ASC networks follows the work of Hu et al. [27] who outlined the way forward to model possible supply chain structures, for example, with four original suppliers as shown in Figure 1.

Figure 1. Possible ASC network with four initial suppliers (adopted from [27]).



Generating all possible combinations of structures creates enormous combinatorial difficulties. Thus, it is proposed here to establish a framework for creating topological classes of assembly supply chains for non-modular and modular ASC networks based on number of initial nodes “i” respecting the following rules:

1. The initial nodes “i” in topological alternatives are allocated to possible tiers  $t_l$  ( $l = 1, \dots, m$ ), ordered from left to right, except the tier  $t_m$ , in which a final assembler is situated. We assume to model ASCs only with one final assembler. In a case when a real assembly process consists of more than one final assembler (for example 3) then it is advisable, for the purpose of the complexity measuring, to split the assembly network into three independent networks.

2. The minimal number of initial nodes “i” in the first tier  $t_l$  equals 2.

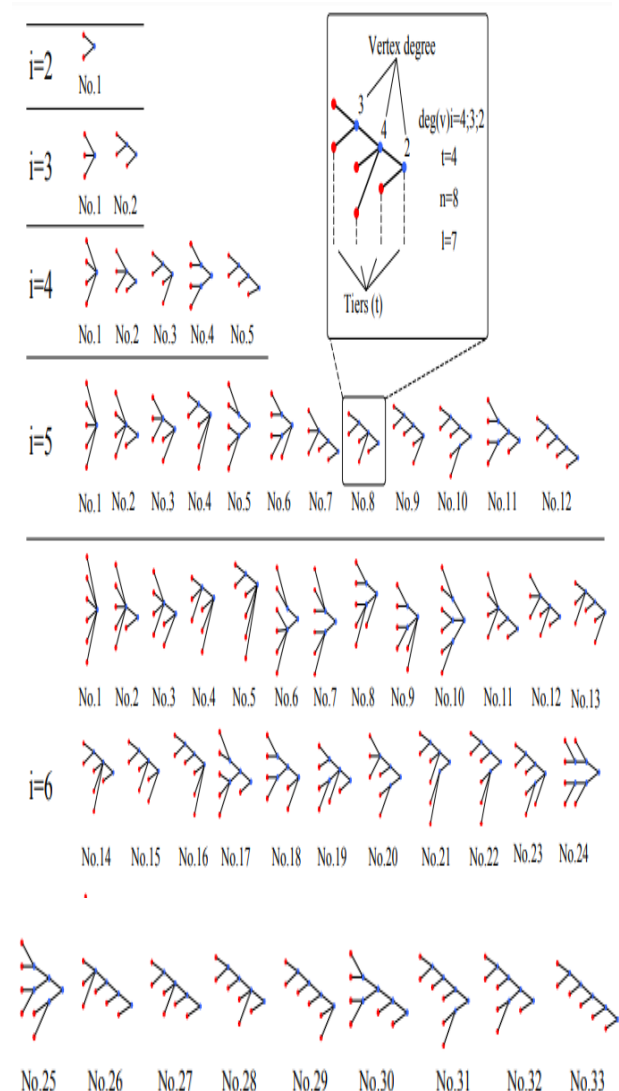
3. In case of non-modular assembly supply chain structure, the number of initial nodes “i” in the most upstream echelon is equal to the number of individual assembly parts or inputs ( $i_n = 1, \dots, r$ ).

Then, all possible structures for given number of initial nodes “i” can be created. An example of generating the sets of structures for the classes with numbers of initial nodes from 2 to 6 is shown in Figure 2.

The numbers of all possible ASC structures for arbitrary class of a network can be determined by the following manner. We first need to calculate the sum of non-repeated combinations for each class

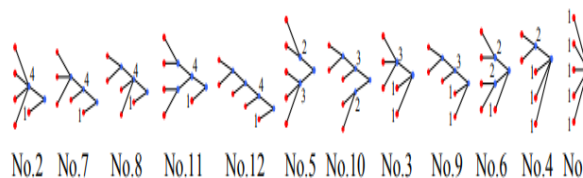
of ASC structures through the so called the Cardinal Number [28]. The individual classes are determined by number of initial nodes “i”. Then, for any integer  $v \geq 2$ , we denote Cardinal Number by  $S(v)$  the finite set consisting of all  $q$ -tuples  $(v_1, \dots, v_q)$  of integers  $v_1, \dots, v_q \geq 2$  with  $v_1 + \dots + v_q \leq v$ , where  $q$  is a non-negative integer

**Figure 2.** Graphical models of the selected classes of ASC structures.



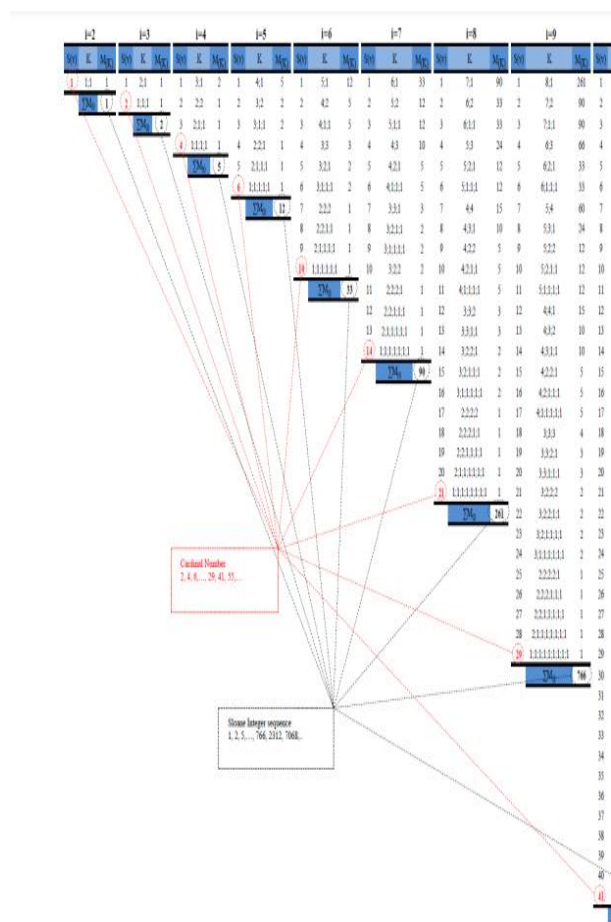
The Cardinal Number  $\#S(v)$  of  $S(v)$  is equal to  $p(v) - 1$ , where  $p(v)$  denotes the number of partition of “v”, which increases quite rapidly with the number of initial nodes “i”. For instance, for  $i = 2, 3, 4, 5, 6, 7, 8, 9, 10$ , the cardinal numbers  $\#S(v)$  are given by 1, 2, 4, 6, 10, 14, 21, 29, 41 (A000065 sequence), respectively [29]. Subsequently, for each non-repeated combination “K”, a multiplication coefficient “M(K)” has to be assigned. The combination “K” is established based on the number of inputs to the final assembler “in” which is situated in tier  $t_m$  (see Figure 3).

**Figure 3.** The transition of graphical ASC networks to the numerical combinations for  $i = 5$ .



Then,  $\sum M_{(i)}$ —the number for all possible combinations of ASC structures for a given class  $i$  obtained. This number is applied in Figure 4.

Figure 4. Determination of total combinations of ASC networks related to the given classes.



A critical step in determining all possible combinations of ASC structures for a given class (starting with a class for  $i = 2$ ) are rules by which we can prescribe a multiplication coefficient “ $M(K)$ ”. In the case when we consider the number of initial nodes equals 2, there is only one numerical combination  $K = (1;1)$  corresponding with appropriate graphical model of assembly supply chain structure, and thus  $M(1;1) = 1$ . Similarly, for each non-repeated numerical combination “ $K$ ” an exact logic rule has to be found. Accordingly we can formulate the following rules:

R1: If the numerical combination “ $K$ ” consists only of numeric characters (digits), assigned by symbol “ $n$ ”,  $n \leq 2$ , e.g.  $K = (2;1)$  or  $K = (2;2;1)$  then  $M(2;1)$  or  $M(2;2;1) = 1$ . R2: If the numerical combination “ $K$ ” consists just of one digit “3” and other digits are  $< 3$ , e.g.,  $K = (3;1)$  or  $(3;2;2)$ , then  $M(3;1)$  or  $M(3;2;2) = 2$ . R3: If the numerical combination “ $K$ ” consists just of one digit “4” and other digits are  $< 3$ , e.g.,  $K = (4;2)$ , then  $M(4;2) = 5$ .

Equally, we could continue to determine multiplication coefficients “ $M_{(K)}$ ” for similar cases when numerical combinations “ $K$ ” consist just of one digit  $\geq 5$  and other digits are  $< 3$  or do not appear respectively. Then we would obtain the following multiplication coefficients:  $M_{(5;1)} = 12$ ;  $M_{(6;1)} = 33$ ;  $M_{(7;1)} = 90$ ;  $M_{(8;1)} = 261$ ; etc.. The multiplication coefficients for the given classes  $\sum M_{(i)}$  in such case, follow the Sloane Integer sequence 1, 2, 5, ..., 261, 766, 2312, 7068, ... (A000669 sequence) [30], and are depicted in Table 1.

Table 1. Determination of all relevant alternatives for structural combinations of ASC networks.

The highest digit of combination set under condition that other digits are $< 3$	Number of alternatives for the given combinations
2	1
3	2
4	5
...	...
8	261
9	766
...	...
17	7,305,788
...	...

For other cases the following rules can be applied:

R4: If the numerical combination “ $K$ ” consists of arbitrary number of non-repeated digits assigned as “ $j, k, l, \dots, z$ ” that are  $\geq 3$  and other digits in the combination are  $< 3$  or do not appear respectively, then the following calculation method can be used:

$$M(j, k, l, \dots, z) = M_j \times M_k \times M_l \times \dots \times M_z, \dots, (1)$$

In order to apply this general rule under conditions specified in R4 the following examples can be shown:  $M(4, 3) = M_4 \times M_3 = 5 \times 2 = 10$  (2)

$$M( ) ( ) ( ) 5;4;3 = M_5 \times M_4 \times M_3 = 12 \times 5 \times 2 = 120 \quad (3)$$

$$M( ) ( ) ( ) ( ) 6;3;2;1 = M_6 \times M_3 \times M_2 \times M_1 = 33 \times 2 \times 1 \times 1 = 66 \quad (4)$$

R5: If the numerical combination “K” consists just of two digits “3” and other digits in the combination are < 3 or do not appear respectively, then  $M(3;3) = 3$ . Calculation of this multiplication coefficient can be formally expressed in this manner:

$$M_{(3;3)} = M_3 + (M_3 - 1) \Rightarrow M_{3,3} = 2 + 1 = 3 \quad (5)$$

R6: If the numerical combination “K” consists just of two digits “4” and other digits in the combination are < 3 or do not appear respectively, then  $M(4;4) = 15$ . Thus,  $M(4;4)$  is computed similarly to Equation (5):

$$M_{(4;4)} = M_4 + (M_4 - 1) + (M_4 - 2 + (M_4 - 3) + (M_4 - 4) \Rightarrow M_{4,4} = 5 + 4 + 3 + 2 + 1 = 15 \quad (6)$$

R7: If the numerical combination “K” consists just of two digits “5” and other digits in the combination are < 3 or do not appear respectively, then  $M(5;5) = 78$  and the multiplication coefficient is computed similarly as Equations (5) and (6):

$$M_{(5;5)} = M_5 + (M_5 - 1) + (M_5 - 2 + (M_5 - 3) + (M_5 - 4) + (M_5 - 5) + (M_5 - 6)$$

$$+ (M_5 - 7) + (M_5 - 8) + (M_5 - 9) + (M_5 - 10) + (M_5 - 11) \quad M_{(5;5)} = 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 78$$

Analogously, we can calculate multiplication coefficients “M(K)” for arbitrary cases when numerical combinations “K” consist just of two digits  $n \geq 3$  and other digits in the combination are < 3 or do not appear respectively. For such cases we can calculate the multiplication coefficients by this equation:

$$M_{(n;n)} = M_{(n)} + (M_{(n)} - 1) + (M_{(n)} - 2) + \dots + [M_{(n)} - (M_{(n)} - 1)]$$

R8: If the numerical combination “K” consists just of three digits “3” and other digits in the combination are < 3 or do not appear respectively, then  $M(3;3;3) = 4$ . Calculation of this multiplication coefficient can be formally expressed in this manner:

$$M_{(3;3;3)} = M_{(3;3)} + (M_{(3;3)} - M_{(3)}) + [M_{(3;3)} - M_{(3)} - (M_{(3)} - 1)] \Rightarrow M_{(3;3;3)} = 3 + (3 - 2) + [3 - 2 - (2 - 1)] = 4$$

R9: If the numerical combination “K” consists just of three digits “4” and other digits are < 3 or do not appear respectively, then  $M(4;4;4) = 15$ . Calculation of this multiplication coefficient can be formally expressed in this manner:

$$\begin{aligned} M_{(4;4;4)} &= M_{(4;4)} + (M_{(4;4)} - M_{(4)}) + \\ &+ [M_{(4;4)} - M_{(4)} - (M_{(4)} - 1)] + \\ &+ [M_{(4;4)} - M_{(4)} - (M_{(4)} - 1) - (M_{(4)} - 2)] + \\ &+ [M_{(4;4)} - M_{(4)} - (M_{(4)} - 1) - (M_{(4)} - 2) - (M_{(4)} - 3)] + \\ &+ [M_{(4;4)} - M_{(4)} - (M_{(4)} - 1) - (M_{(4)} - 2) - (M_{(4)} - 3) - (M_{(4)} - 4)] \end{aligned}$$

$$\begin{aligned} M_{(4;4;4)} &= 15 + (15 - 5) + [15 - 5 - (5 - 1)] + [15 - 5 - (5 - 1) - (5 - 2)] + \\ &+ [15 - 5 - (5 - 1) - (5 - 2) - (5 - 3)] + [15 - 5 - (5 - 1) - (5 - 2) - (5 - 3) - (5 - 4)] = 35 \end{aligned}$$

R10: If the numerical combination “K” consists just of three digits “5” and other digits in the combination are < 3 or do not appear respectively, then  $M(5;5;5) = 78$ . Calculation of this multiplication coefficient can be formally expressed in this manner:

$$\begin{aligned} M_{(5;5;5)} &= M_{(5;5)} + (M_{(5;5)} - M_{(5)}) + \\ &+ [M_{(5;5)} - M_{(5)} - (M_{(5)} - 1)] + \\ &+ [M_{(5;5)} - M_{(5)} - (M_{(5)} - 1) - (M_{(5)} - 2)] + \\ &+ [M_{(5;5)} - M_{(5)} - (M_{(5)} - 1) - (M_{(5)} - 2) - (M_{(5)} - 3)] + \\ &+ [M_{(5;5)} - M_{(5)} - (M_{(5)} - 1) - (M_{(5)} - 2) - (M_{(5)} - 3) - (M_{(5)} - 4)] + \\ &+ [M_{(5;5)} - M_{(5)} - (M_{(5)} - 1) - (M_{(5)} - 2) - (M_{(5)} - 3) - (M_{(5)} - 4) - (M_{(5)} - 5)] + \\ &+ [M_{(5;5)} - M_{(5)} - (M_{(5)} - 1) - (M_{(5)} - 2) - (M_{(5)} - 3) - (M_{(5)} - 4) - (M_{(5)} - 5) - (M_{(5)} - 6)] + \\ &+ [M_{(5;5)} - M_{(5)} - (M_{(5)} - 1) - (M_{(5)} - 2) - (M_{(5)} - 3) - (M_{(5)} - 4) - (M_{(5)} - 5) - (M_{(5)} - 6) - \dots - (M_{(5)} - 11)] \end{aligned}$$

$$M_{(5;5;5)} = 78 + 66 + 55 + 45 + 36 + 28 + 21 + 15 + 10 + 6 + 3 + 1 = 364$$

A general rule to calculate the multiplication coefficients “M(K)” for arbitrary cases (when



numerical combinations “K” consist just of three digits  $n \geq 3$  and other digits in the combination are  $< 3$  or do not appear respectively) can be derived using the previous rules R8, R9 and R10 a formally can be expressed as:

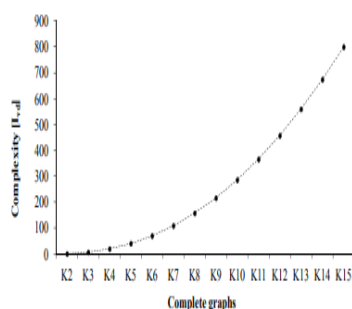
$$M_{(n;n)} = M_{(n;n)} + (M_{(n;n)} - M_{(n)}) + \\ + [M_{(n;n)} - M_{(n)} - (M_{(n)} - 1)] + \\ + [M_{(n;n)} - M_{(n)} - (M_{(n)} - 1) - (M_{(n)} - 2)] + \dots + \\ + \{M_{(n;n)} - M_{(n)} - (M_{(n)} - 1) - (M_{(n)} - 2) - \dots - [M_{(n)} - (M_{(n)} - 1)]\}$$

Obviously, there are other specific cases of numerical combinations for which multiplication coefficients can be formulated in exact terms.

### The Concept of Quantitative Complexity Scale for ASC Networks

Basically, the comparison of complexity is of a relative and subjective nature. It is also clear that through a relative complexity metric we can compare the complexity of the existing configuration against the simplest or/and the most complex one from the same class of ASC network. Perhaps, the most important feature of the relative complexity metric is that we can generalize it to other areas [35]. Accordingly, when we apply this complexity measure for the complete graphs with  $v(v-1)/2$  edges we can get upper bounds for configuration complexity of any ASC structure with a given number of vertices. Obtained upper bounds derived from complexity values of selected complete graphs are shown in Figure 7.

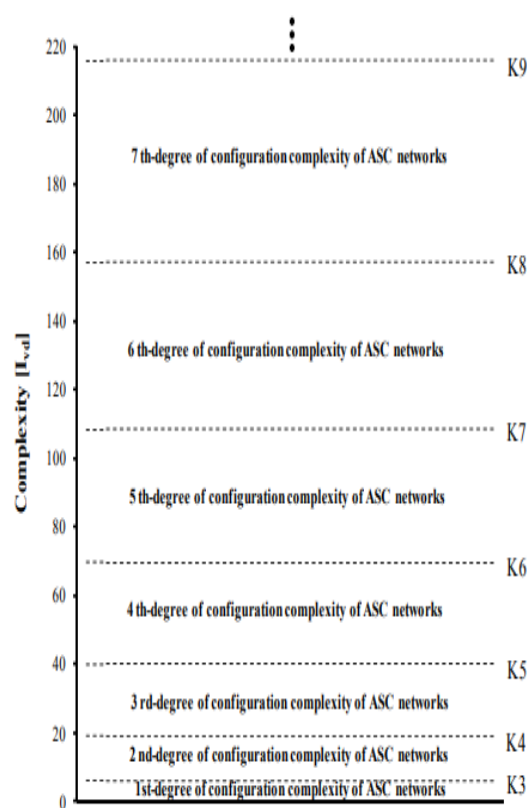
Figure 7. Graph of the complexity measures for the selected complete graphs.



When considering the fact that obtained complexity values for the complete graphs grow larger and larger, while complexity values of ASC structures

for ascending ordered classes grow smaller and smaller it gives a realistic chance to establish quantitative complexity degrees of ASC networks. Under this assumption, arbitrary ASC networks can be categorized into quantitative configuration complexity degrees that are shown in Figure 8. In such case, the actual question arises regarding how many degrees of structural complexity are really needed to comprise all ASCs that we know exists. The seven-degree scale of structural complexity is based on inductive reasoning. For example, upper bound for configuration complexity of ASC networks with  $i = 10$  equals 40.04. Indeed, it is very presumable that practically all realistic ASC networks wouldn't reach higher structural complexity than 216 what presents structural complexity for K9. However, in this context, it is necessary to take under consideration a relation between complexity and usability [36]. In this case it would be needed to estimate an optimal degree of structural complexity under when the usability of ASC networks is critical for its success.

Figure 8. Proposed quantitative complexity degrees.



### Conclusions

The main contributions of this paper consist of the following four aspects:

(1) A new exact framework for creating topological classes of ASC networks is developed. This methodological framework enables one to determine all relevant topological graphs for any class of ASC structure. The usefulness of such a framework is especially notable in cases when it is necessary to apply relative complexity metrics to compare the complexity of the existing configuration against the simplest or/and the most complex one.

(2) In order to parameterize properties of vertices of the ASC networks, an efficient method to identify total number of the graphs with non-repeated sets of vertex degrees structure is presented. The determination of the non-repeated sets of vertex degrees structure (for selected classes of ASC networks are described in Figure 5) shows that the total numbers of such graphs follows the Omar integer sequence [37], with the first number omitted.

(3) The Vertex degree index was applied to a new area of configuration complexity.

(4) The quantitative object-oriented model for defining degrees of configuration complexity of ASC networks was outlined.

The proposed approach to relative complexity assessment may easily be applied at the initial design stages as well as in decision-making process along with other important considerations such as operational complexity issues. However, this research path requires further independent research to confirm this preliminary results and proposals.

**Acknowledgments:** This work has been supported by the financing from KEGA grant - Cultural and Education Grant Agency of Ministry of School, Science, Research and Sport of Slovak Republic.

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